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A State Space Stand Model for Pinus radiata in Chile

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STATE SPACE, SIMULATION, DIFFERENTIAL EQUATIONS, LIKELIHOOD

1. Brief story of the state space approach

The state space approach is a very powerful general methodology, that embraces a whole class of dynamic processes models.

Shumway (1988) has indicated that it was first introduced in relation to aerospace research by Kalman (1960), Kalman and Bucy (1961).

General concepts can be found in the textbook of Wiberg (1971).

Shumway cites early applications to economics (Harrison and Stevens, 1976). The methodology has been developed and applied to several dynamic processes lately.

From the best of my knowledge, the first application of these concepts in forestry was done by García (1974), included in a publication of the chilean Forest Research Institute.

There are interesting applications in medicine (Jones, 1984), and soil analysis (Shumway, 1985).

2. Basic concepts as applied to stand simulation

To explain this methodology in simple words, the stand may be considered a dynamic system, whose state at any given moment t, p(t), can be represented by adequately chosen state variables. This state evolves over time in a space, which embraces all possible states the system may take.

Taking advantage of notation and concepts for this approach in forestry, (García, 1974), the stand is considered a system. This means it is a collection of elements, the trees, relations between them, competition for light, water and nutrients, and interactions with the environment.

The state variables constitute the state vector of the system:

$$p(t) = [\ x1(t),\ x2(t)\ \ldots\ ] \qquad (1)$$

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The future state of the stand depend only on the present state and an eventual control vector, representing management interventions,

$$p(t+1) = F1 [p(t) , u(t)] \qquad (2)$$

where u(t) is the input or control vector.

The most common interventions in even-aged management are thinnings, pruning and fertilizing.

Thinnings can be considered as instantaneous state changes made between growing periods, and therefore, left out of u(t).

Pruning represent a vertical change in the crowns of pruned trees, and if the crown is adequately represented in the state of the stand, this silvicultural practice can be considered as an instantaneous change also. Since it happens between growing periods, it may be left out of u(t) in the same way.

Modeling fertilization is a more complex task. Strictly speaking, it is a genuine control variable that acts throughout the growing period. Fertilizing effect in reality is a gradual but temporal change in site index, that will attain a climax effect some years after application, and then begin to decline again.

In the model it is treated as a temporal change in site index, following a cycle as explained above. Special studies in the field are advisable to define the form and duration of the cycle.

2.1 Transition between two consecutive states

The process of transiting from one state to the following may be written as:

$$dp = p(t+1) - p(t) = F2(p(t)) \qquad (3)$$

which is another function of the present state. It is the present state of the stand that will determine how it will evolve.

If the variation is referred to the time interval $dt = t_1 - t$, observations measured along unequal spacing in time are homogenized, yielding:

$$\frac{dp}{dt} = F3 (p(t)) \qquad (4)$$

which becomes a differential equation when lim dt ---> 0.

The processes in which the future state are determined based only in the present state are known as Markovian processes.

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Specifically, in the case of Pinus radiata in Chile, (De la Maza, 1978) and Pinus patula in Mozambique and Zimbabwe (de la Maza, 1986), the state vector:

$$p(t) = [\ G(t),\ N(t),\ H(t),\ S\ ] \qquad (5)$$

was found adequate. G(t) is basal area, N(t) is the number of stems both per hectare, H(t) is mean dominant height of the stand, at age t, represents a reasonably adequate starting state vector for even-aged stands of these species.

2.2 Dealing with recent thinnings and prunings

Two stands with different number of stems will have different mean spacings, and a different competition intensity is expected.

Even two stands with the same number of stems but different basal area are expected to differ in their behavior.

Moreover, two stands with same number of trees, the same basal area, but different dominant mean heights, will have diverse relative spacing (Beekhuis, 1966), and therefore different radiation and growing conditions.

Prior models (De la Maza (1978, 1986), (García (1978), considered a state vector as in (5), leaving out crown. However it has proven to be a good approximation to reality.

However, a recently thinned and pruned stand with the same three state variables and site index, will be growing under different conditions than one that hasn´t been altered. Moreover, pruned trees grow less than unpruned trees, also called followers, when left uncut for a period.

In the specific case of an even-aged pine stand, however, one might think that growth cannot be equivalent immediately after thinning or pruning to some 2 or 3 years later, when the canopy begins to close again, or later on, when canopy is almost completely closed.

Photosynthetic surface after thinning or/and pruning is affected, and will be different from that of a stand of the same age without having been thinned or/and pruned recently. The recently intervened stand cannot immediately make use of new available space.

On the other hand, free spaces generated horizontally by thinnings and vertically by prunings at crown level, imply that site occupancy is not complete.

In this fashion, an additional state variable for representing

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crown may be able to capture changes recently produced in the canopy and branches, that weren´t included in the original state vector.

The part of the tree most rapidly affected by interventions that alter light availability in the stand is the crown. And the crown is the factory where photosynthesis is carried out. Therefore it is determinant in competition and growth.

The state vector turns to:

$$p(t) = [\ G(t),\ N(t),\ C(t),\ H(t)\ ] \qquad (6)$$

where C(t) states for a representation of crowns. The sum of the basal area at the base of live crowns was selected for this purpose, using ideas of the pipe model (Waring et al., 1982).

Explicit presence of site index S in p(t) is unnecessary, since it is implicitly represented in (6), through dominant height H the stand reach at age t, and in site index equation.

3. Generalizing state vector

A more general vector with a one to one correspondence to the state vector in (6) can be set up as follows:

$$\underline{X}(t) = [\ X_1(t),\ X_2(t),\ X_3(t),\ H(t)] \qquad (7)$$

where each $X_i(t)$ is generated as:

$$X_1(t) = G^{\alpha 1}\ N^{\beta 1}\ C^{\Gamma 1}\ H^{\phi 1} \qquad (8)$$

Choosing adequate exponents α_1, β_1, Γ_1, ϕ_1 a better fit of equations to data can be obtained. The key is to choose variables X with a biological meaning. For instance, the mean relative spacing is generated when $\alpha=-0.5$, $\beta=-1$, $\Gamma=0$ and $\phi=0$.

The original state vector p(t) in (6) is a particular case of the generalized vector $\underline{X}(t)$, in which $\alpha 1=1$, $\beta 1=0$, $\Gamma 1=0$, $\phi 1=0$, $\alpha 2=0$, $\beta 2=1$, $\Gamma 2=0$, $\phi 2=0$, $\alpha 3=0$, $\beta 3=0$, $\Gamma 3=0$, $\phi 3=1$. See Fig #1.

4. Model structure

Up to this point the model is configured as a system of differential equations of the form:

$$dX_1/dt = a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + b_1t + c_1 \qquad (9)$$

$$dX_2/dt = a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + b_2t + c_2 \qquad (10)$$

$$dX_3/dt = a_{31}X_1 + a_{32}X_2 + a_{33}X_3 + b_3t + c_3 \qquad (11)$$

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or

$$\frac{d\underline{X}}{dt} = A\underline{X} + \underline{b}t + \underline{c} \qquad (12)$$

FIG #1 Alternative X transformed variables

i	α	β	Γ	ø	X_1 obtained	Meaning
1	1	0	0	0	G	Stand basal area
2	0	1	0	0	N	Stand number of stems
3	1	-1	0	0	G/N	≈ to mean basal area
4	0.5	-0.5	0	0	√G/√N	≈ to quadratic mean DBH
5	0	-0.5	0	0	1/√N	≈ to mean spacing
6	0	-0.5	0	-1	1/(√N*H)	≈ to mean relative space.
7	0.5	0.5	0	1	√G*√N*H	≈ to stems surface
8	1	0	0	1	G*H	≈ to stand total volume
9	1	-1	0	1	(G*H)/N	≈ to mean volume
10	0	-0.5	0	0	1/√G	≈ to mean relat. space DBH
11	0	0	1	0	C	≈ total crowns of stand
12	0	1	0.5	0	N*√C	≈ crown base sum of DBH
13	-1	.5	0	0	√B/N	≈ inverse sp. to bas. area
14	-1	0	1	0	C/B	≈ crown relat to bas. area
15	0	0	1	-1	C/H	≈ crown relat to D. height

System (12) is very simple and conventional in the parameter estimation process, but qualitatively different and substantially more powerful to describe the behavior of the stand than conventional models. Instead of fitting a curve that passes through the middle of a plot of points, it is like a field of curves fitted to a set of slopes representing rates of changes observed in the experiments.

Integration of equation (12) yields the transition equation (13) which represents movement from one state to another in time.

For accuracy, transition functions must be developed locally, and for representing the behavior of continuous-time variating systems, as a forest stand, it must take the form of a differential equation, as in (12).

$$\underline{X}(t) = \underline{F}(\underline{X}(t_o), \underline{u}, t-t_o) \qquad (13)$$

Agreeing with García (1978), the transition function $\underline{F}$ must satisfy what he called natural conditions, which are:

a) consistency:

$$\underline{F}(\underline{X}(t),0) = \underline{X}(t) \qquad (14)$$

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for all $\underline{X}(t)$ and admissible input functions $\underline{u}$. Then, if $t-t_o = 0$, transition is null and state remain unchanged.

b) composition or semigroup property:

$$\underline{F}(\underline{F}(\underline{X}(to),t_1-t_o),t_2-t_1) = \underline{F}(\underline{X}(t_o),t_2-t_o) \qquad (15)$$

It means that a projection from t_o to t_1 and then from t_1 to t_2 must be the same as a direct projection from t_o to t_2.

c) causality: if a control function $\underline{u}$ is considered,

$$\underline{F}(\underline{X}(t_o),\underline{u}1,t_1-t_o) = \underline{F}(\underline{X}(t_o),\underline{u}2,t_1-t_o) \qquad (16)$$

for all $\underline{u}1 = \underline{u}2$. If local transition functions are linear as in (12), additional advantages are obtained, such as analytical integration possibility, and ease in parameter estimation.

If input $\underline{u}$ is left out of the growing period, (thinnings and prunings), the transition function takes the simpler form

$$\underline{X}(t) = \underline{F}(\underline{X}(t_o),\ t-t_o) \qquad (17)$$

4.1 Treatment of seasonal growth and uneven periodical measurements

It is well known that measurement intervals in permanent plots are not always equal. In the system of equations above, this would have been solved with the reference of the increments to time intervals. But it is a fact that growth isn´t homogeneous over time in a year, due to the season´s varying temperatures and general climate conditions.

If all measurements were done close to the dormant season, no problem would arise. A growth explosion has been reported from independent researchers in Spring (Cortés, 1969). So a delay in measuring in this season would introduce bias.

As this author reported in his thesis work, growth was found to be closely related to mean temperature and its distribution over time, starting at about 10 C°.

A way of correcting this problem is to develop a table of monthly growth over year, and to change data measured in the field by some constant derived from this table.

This method has some problems, such us:

- arbitrariness in the constants used, or high cost of defining them separately by zones. A specifically related experiment would be necessary for this

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- necessity of correcting each variable by a different table, because for example, diameter and height growth have different patterns over year.

- problems in correcting diameter, since the mean diameter is altered by management practices. This is not the case for height.

- incertitude of using data that wasn´t actually measured in the field, but altered by a somewhat arbitrary criterion.

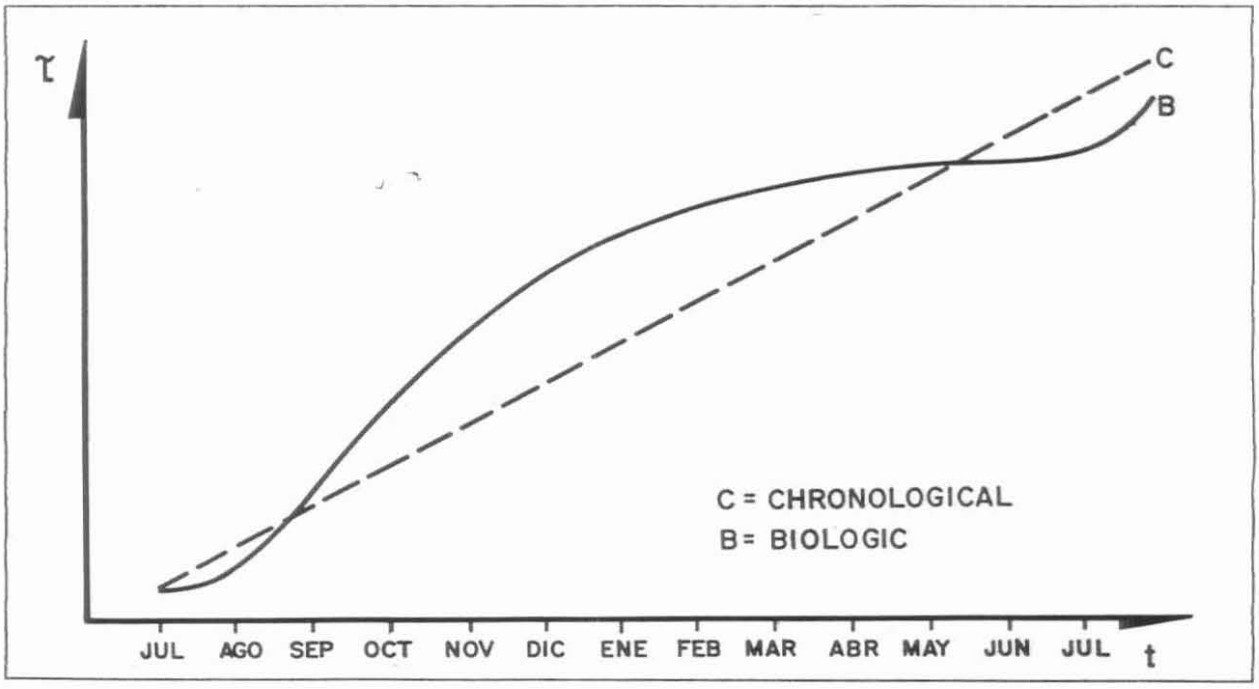
A more helpful methodology for compensating this is to refer increments of state variables to increments in dominant height.

Dominant height monthly growth curve according to year has its own pattern, not affected by thinnings as is mean diameter or basal area. This growth curve is also closely related to mean temperature and temperature distribution over the year.

Another advantage is that the shape of this growth curve is particular for each forest site, so correction is automatically different for different forest sites.

Mean dominant height growth is then a different dimension of time, a kind of "biological time". So if increments are measured in the field at different intervals, even exceeding the dormant season, they are made comparable.

FIG #2 Chronological and biologic time



On the other hand, chronological time is linear, and does not follow season variations in growth. A one month delay in spring is not equivalent to a one month delay in winter in biologic terms, but they are equal in chronological terms. FIG #2.

Nevertheless, the problem of differences in growth among variables may require further attention. In fact basal area growth follow its own pattern, following climatic variations, but showing some delay with respect to height (Mead, 1990).

Differences in seasonal growth are greater in the four months, November, December, January and February. So better accuracy is possible leaving out plots measured in these months.

Based on the analysis above, refering growth to a biologic time τ based on height growth, yields the following system:

$$\frac{d\underline{X}}{d\tau} = A\,\underline{X} + \underline{b}\tau + \underline{c} \qquad (18)$$

where τ states for biologic time. Its relation to chronological time is of the form:

$$\tau = g(t, S) \qquad (19)$$

which in fact is the site index curve of the site, where t is age in years and S is site index in meters.

This approach has been used before implicitly by Beekhuis (1966) and explicitly by García (1978) and De la Maza, (1978, 1986).

5. Solution of the system of differential equations

The general solution of the system above is (Roxin, 1968):

$$\underline{X}(\tau) = e^{A(\tau-\tau_0)}\,\underline{X}_0 + \int_{\tau_0}^{\tau} e^{A(\tau-s)}\,\underline{b}s\,ds + \int_{\tau_0}^{\tau} e^{A(\tau-s)}\,\underline{c}\,ds \qquad (20)$$

The explicit expression of (20) can be obtained using eigenvalue methods on the characteristic equation of matrix A.

At the beginning of the simulation process, initial chronological time t_0 and final chronological time t are converted to the transformed time τ_0 and τ, then projection is performed between τ_0 and τ, using an explicit form of equation (20).

At the end, τ may be converted back to t, for refering reports to chronological time.

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Backward transformation is needed from vector $\underline{X}(\tau)$ to $p(\tau)$, using the inverse of equation (8).

6. Computation procedure

A series of arithmetic operations and decisions must be performed, prior to formulating the final prototype of a working model.

Starting data are aggregated and validated stand state variable observations as defined in vector p(t) in (6). From this data the following computations and decisions can be made:

- form the models and compute series of independent variables X
- compute $\underline{Y}$ variables (untransformed derivatives)
- compute transformed derivatives $\underline{Z}$
- compute and check existence of a not null Jacobian
- parameter estimation in the regression of vector $\underline{Z}$ on $\underline{X}$ and τ, as in equation (25)
- perform goodness of fit test for each equation
- perform goodness of fit test for the system as a whole
- determine existence of an asymptotic stable solution to the system of differential equations
- select best model
- compute solution that will be used in simulation tests

Taking every different subset of 3 among the 15 X_1 variables defined, as in FIG #1, yield 455 alternative models.

In computing derivatives, as each X is:

$$X = G^{\alpha 2} N^{\beta 2} C^{\Gamma 2} H^{\varnothing 2} \tag{21}$$

taking derivative

$$\frac{dX}{d\tau} = \frac{d(G^{\alpha} N^{\beta} C^{\Gamma} H^{\varnothing})}{d\tau} \tag{22}$$

yields

$$\frac{dX}{d\tau} = \alpha G^{\alpha-1} N^{\beta} C^{\Gamma} H^{\varnothing} Y1 + \beta G^{\alpha} N^{\beta-1} C^{\Gamma} H^{\varnothing} Y2 + \Gamma G^{\alpha} N^{\beta} C^{\Gamma-1} H^{\varnothing} Y3 + \varnothing G^{\alpha} N^{\beta} C^{\Gamma} H^{\varnothing-1} \tag{23}$$

Approximation is done by evaluating the expression above in the middle point of the interval (slope in the middle point).

If the Jacobian of the transformation is null, it means the model isn't a one to one transformation, thus it does not allow recovering the original $p(\tau)$ from the projected $\underline{X}(\tau)$. Computing

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time is saved checking this as soon as possible. The Jacobian is:

$$J = \left| \frac{\delta \underline{Z}}{\delta \underline{Y}} \right|_{+} \qquad (24)$$

7. Parameter estimation and model selection

Assumptions are made of linearity in least squares regression parameter estimation.

Violations to this assumptions must be corrected. Some are outlined in detail later.

The process of parameter estimation follows the sequence:

a) Using program REG3.EXE generate models as in equation (12), compute multivariate linear regression by OLS, getting 15 regression parameters estimates for each of the 455 models:

$$\underline{\beta} = [\underline{X}'\underline{X}]^{-1}\ \underline{X}'\underline{Z} \qquad (25)$$

b) Complete the process of generating and selecting models described above. Then choose the two or three best models, check for violation of assumptions, and compute parameters by a corrected process.

7.1 Goodness of fit test of the system as a whole

Likelihood is the probability of the model in estimating the actually observed values for the multivariate $\underline{Z}$, given $\underline{X}$ and H.

Any violation of the assumptions stated above will impact the Likelihood of the system as a whole. So it is necessary to maximize likelihood of the system, estimating parameters simultaneously, and see if any gain in Likelihood is obtained.

A special optimization program is used for this final process. Good starting values must be supplied. OLS values are used for this purpose. Gain will depend on error and general data structure.

The multivariate likelihood function is the objective to be optimized. It is based on the distribution of $\underline{Z}$, supposed to be multivariate normal. Probability density for one observation is:

$$f(\underline{Z}_1) = |\Sigma|^{-0.5}\ (2\pi)^{-0.5p}\ \exp[-0.5(\underline{Z}_1 - \hat{\underline{Z}}_1)'\Sigma^{-1}(\underline{Z}_1 - \hat{\underline{Z}}_1)] \qquad (26)$$

where Σ is the covariance matrix, with inverse Σ^{-1}, and element $\sigma_{1,j}$. $\underline{Z}_1$ is the regression estimation of $\underline{Z}_1$. (i=1,2, ...n), (j=1,2,...m).

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Likelihood for $\underline{Z}$ is then:

$$L(\underline{Z}) = \prod_{i=1}^{n} f(\underline{Z}_i) \qquad (27)$$

As each multivariate $\underline{Z}$ is different among models, they are not directly comparable. Comparison is possible if density is expressed in terms of the untransformed vector $\underline{Y}$. This is:

$$f(\underline{Y}_i) = |\Sigma|^{-0.5}(2\pi)^{-0.5p} J_i \exp[-0.5(\underline{Z}_i - \hat{\underline{Z}}_i)'\Sigma^{-1}(\underline{Z}_i - \hat{\underline{Z}}_i)] \qquad (28)$$

where J_i is the Jacobian of the transformation, as in (24). Then likelihood for $\underline{Y}$ is:

$$L(\underline{Y}) = \left(\prod_{i=1}^{n} J_i\right) |\Sigma|^{-0.5n} (2\pi)^{-0.5np} \exp(-0.5\ Q) \qquad (29)$$

where Q is the same for all models (De la Maza, 1978), (Arner & Seegrist, 1980).

In equation (29), Σ is replaced for its maximum likelihood estimator, which is V/n, $[v_{i,j}]/n$.

Only the part of Likelihood which differs from model to model is interesting for comparison purposes, so constant parts in (29) may be left aside.

For avoiding overflow or underflow in the computer, an index of maximum likelihood is used instead of the complete expression of it. Thus, for computation convenience, taking nth root of the Likelihood of $\underline{Y}$ in $\underline{Z}$, and powering the expression to -1, yields:

$$L(\underline{Y}) = \frac{|V|^{0.5} (2\pi)^{0.5p} [\exp(-0.5np)]^{-1/n}}{(n-r)^{0.5} \left(\prod_{i=1}^{n} J_i\right)} \qquad (30)$$

Eliminating the constant parts, and taking logarithms, the comparison index is:

$$\mathrm{Ln}(L(\underline{Y})) = 0.5\ \mathrm{Ln}|V| - 1/n \sum_{i=1}^{n} \mathrm{Ln}(J_i) \qquad (31)$$

which is actually computed.

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7.2 Checking asymptotic stability of solutions

Transformations are used to find an explicit operative expression for the solution of the system of differential equations, starting with the characteristic equation of matrix A, in the general solution of equation (12):

$$|A - LI| = 0 \qquad (32)$$

First the eigenvalues L_j of A are found by expanding the determinant in (32), where a polynomial of jth degree is found.

Asymptotic stability is attained if in this polynomial all the real parts of the solutions are negative (Kiseliov et al, 1979).

All these calculations are performed by the program REG3.EXE. If the solution is found not to be asymptotically stable, the model is discarded, and the process continues with the next model.

Linearly independent eigenvectors v_j associated with these eigenvalues are found using:

$$(A - LI)\underline{y} = 0 \qquad (33)$$

Then transformations are performed in equation (20) on the exponential matrices, to allow an explicit solution. The transformations are of the form:

$$e^{AT} = P\ [L]\ P^{-1} \qquad (34)$$

where P is a matrix formed with the eigenvectors obtained in (33) and [L] is a diagonal exponential matrix on the corresponding L´s obtained in (32).

7.4 Selection of the best model

The best model so far will be the one among all with a non-null Jacobian, good multiple correlation in each equation, passed all conventional regression tests stated before, with an asymptotic stable solution, that has the highest Likelihood.

After all this, the solution constants are logged into the actual simulator program, called RODAL.EXE, that allows run and test the model dynamically, for any management regime.

This simulator frame has powerful capabilities, such as graphics outputs, commercial volume yield functions, and economic analysis under several criteria, to ease the study of the behavior of the model.

RODAL.EXE is available at the Chilean Forest Research Institute (Instituto Forestal) CAS 3085, Santiago Chile.

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8. Further improvement in parameter estimation

If heterocedasticity is detected, it may be corrected using weights based on the variance-covariance matrix. In this case the parameters for each equation would be:

$$\hat{\underline{\beta}} = [\underline{X}'V^{-1}\underline{X}]^{-1}\ \underline{X}'V^{-1}\underline{Z} \qquad (35)$$

which correspond to the generalized least squares estimates GLS.

If one independent variable is linearly dependent on a group of others (multicollinear), it is discarded. This is detected through eigenvalues of the X´X matrix.

From the simultaneity point of view, according to Borders (1989), systems of related equations may be classified in three groups:

1) seemingly unrelated equations
2) simultaneous equations
3) recursive equations

If there is no simultaneity and the model is linear, parameter estimation is equivalent to the seemingly unrelated case.
If simultaneity exists, an improvement on the likelihood will be obtained, and different parameters estimates also.
The second case assumes that cross correlation exists among error of the various equations.

When fitted to data, no evident change was obtained starting from the first case.

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COMPUTER PROGRAMS MENTIONED

REG3.EXE is a special program for multivariate regression of differential equations. (De la Maza, 1986-1990).

RODAL.EXE is the computer structure under which runs the model. (De la Maza, 1986-1990).

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